

Fig.2. Thresholds vs. α for $\lambda=0.5, \theta=0.3, \mu=1, \mu_0=0.6, C=2, R=50$

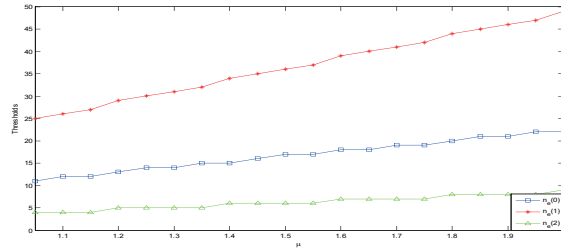


Fig.3. Thresholds vs. μ for $\lambda=0.5, \alpha=0.05, \theta=0.3, \mu_0=0.6, C=2, R=50$

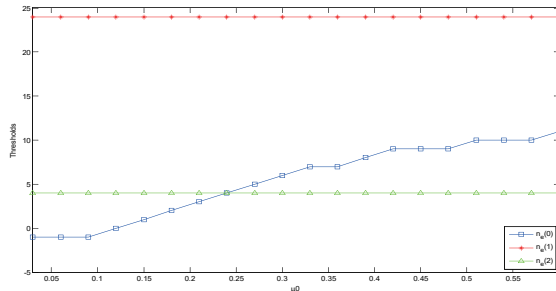


Fig.4. Thresholds vs. μ_0 for $\lambda=0.5, \alpha=0.05, \theta=0.3, \mu=1, C=2, R=50$

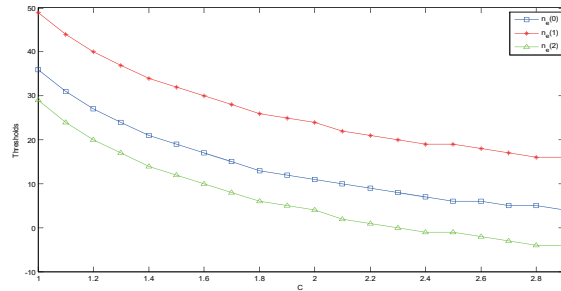


Fig.5. Thresholds vs. C for $\lambda=0.5, \alpha=0.05, \theta=0.3, \mu=1, C=0.6, R=50$

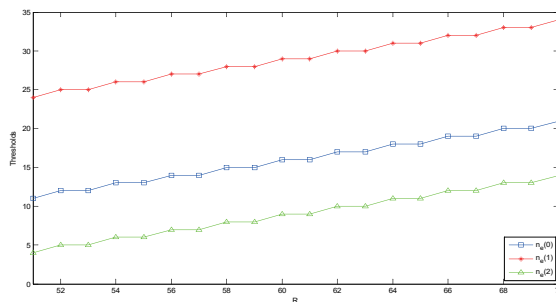


Fig.6. Thresholds vs. R for $\lambda=0.5, \alpha=0.05, \theta=0.3, \mu=1, \mu_0=0.6, C=2$

In Fig.1 we observe that the threshold $n_e(0)$ is monotonically increasing function of transition rate θ . This is because the customers are served at a faster service rate so as to decrease their overall delay, when the server switches to the normal working state from working vacation state more frequently. Therefore, they are more likely to enter . This is expected from Theorem 1 that $n_e(1)$ and $n_e(2)$ remain constant since θ is irrelevant to the customer’s decision. Fig.2 we observe the threshold $n_e(2)$ is monotonically increasing function of

setup rate α . This is because when the server converts to the normal working level more faster from the setup period, the customers have more chances to be served at faster service rate. Thus they prefer to enter. $n_e(0)$ and $n_e(1)$ remain constant since setup rate is irrelevant to customer's decision. Fig.3 depict the equilibrium thresholds versus service rates μ . It is intuitive that an arriving customer is more likely to enter when the server can serve more customers per time unit. Fig.4 shows that the threshold $n_e(0)$ is monotonically increasing function of service rates μ_0 . Comparing Fig.5 and Fig.6 we know that the higher reward R the customers gain from service, the higher the waiting cost they can afford. So the thresholds increase with R . However, if waiting costs customers more money, they may be not willing to wait for service avoiding paying too much. the thresholds decrease with C .

4. Conclusions

In this paper we considered the problem of analyzing customer strategic behavior, in the fully observable M/M/1 queue with working vacations and setup period, where customers decide whether to join the queue or balk upon arrival. We list numerical examples to study the behavior of customers under varying parameters of the system.

References

- [1] Burnetas, A., & Economou, A.: Equilibrium customer strategies in a single server Markovian queue with setup times. *Queueing Systems* 56(2007)213-228.
- [2] Edelson, N.M., Hildebrand, K.: Congestion tolls for Poisson queueing processes, *Econometrica* 43 (1975) 81-92.
- [3] Guo, P., Hassin, R.: Strategic behavior and social optimization in markovian vacation queues. *Operations Research* 59(2011)986-997
- [4] Hassin, R., Haviv, M.: Equilibrium threshold strategies: the case of queues with priorities. *Operations Research* 45 (1997) 966-973.
- [5] Jain, M. & Upadhyaya, S. (2011). Synchronous working vacation policy for finite-buffer multiserver queueing system. *Applied Mathematics and Computation*, 217, 9916–9932.
- [6] Liu, W. & Wang, J. & Li, J. (2012). Equilibrium threshold strategies in observable queueing systems under single vacation policy. *Appl. Math. Model.*, 36, 6186-6202.
- [7] Naor, P.: The regulation of queue size by levying tolls, *Econometrica* 37(1969)15-24.
- [8] Servi, L.D. & Finn, S. G. (2002). M/ M /1 queues with working vacations (M/ M/1/WV). *Performance Evaluation*, 50, 41–52.
- [9] Sun, W. & Guo, P. & Tian, N. (2010). Equilibrium threshold strategies in observable queueing systems with setup/closedown times. *Central European Journal of Operations Research*, 18, 241-268.
- [10] Tian, N. & Zhao, X. & Wang, K. (2008). The M /M /1 queue with single working vacation. *International Journal of Information and Management Science*, 19, 621-634.
- [11] Wang, H. , Xu, X. , & Wang, S. (2019). Equilibrium customers strategies in the markovian working vacation queue with setup times. *International journal of computing science and mathematics*, 10(5), 443-458.

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