

# Effective Calculation Methods of Reference Sound Velocity in Shallow Sea Environment

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**Abstract:** The conventional underwater passive direction finding method estimates the target angle through beamforming. The reference sound velocity in horizontal linear array beamforming should use the phase velocity of sound propagation. In passive direction finding, because the target distance is unknown, there is a deviation between the reference sound velocity selected in the target angle estimation and the phase velocity, which affects the direction finding accuracy. In this paper, two methods for calculating the reference speed of sound are presented: the normal amplitude weighting method and the beamforming method. The numerical simulation results under shallow sea propagation condition show that the reference sound velocity calculated by the two methods has the same trend as the distance between target and array.

**Keywords:** Reference Sound Velocity; Phase Velocity

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## 1. Introduction

Array signal processing is a technology that arranges multiple sensors in space in a certain order to form a sensor array, acquires signals in space, and performs signal enhancement by means of signal processing using the received data and estimates its parameters, and has very wide and important applications in many fields such as sonar, radar, and seismic. Beamforming technique and spatial spectrum estimation are two important research problems in array signal processing. Beamforming technology refers to the weighted processing of the received signals of each element in the receiving array, so as to achieve the purpose of enhancing the desired signal and suppressing noise and interference signals. Spatial spectrum estimation, also known as direction of arrival estimation, is used to determine the Angle at which the target signal reaches the reference element of the receiving array in space.

Specifically in the field of hydroacoustics, the underwater direction of arrival estimation is widely used in military and civil areas, such as underwater target positioning and tracking, ocean resources exploration and development. Therefore, it is of great practical significance to improve the accuracy of target Angle estimation. Sonar is a tool for underwater target detection and positioning. Sonar detection and positioning is a technology to determine the target position based on the sound field information of the target received by the hydrophone array. The variation of sound velocity in seawater determines the propagation distance and path of sound waves. The accurate measurement of sound velocity is of great significance for

underwater detection targets. The measurement methods of sea water sound velocity are generally divided into direct measurement and indirect measurement. The direct measurement method uses the sound velocity profiler for measurement, which has high measurement accuracy but low measurement efficiency and high cost. The indirect measurement method is based on the empirical formula of the speed of sound to calculate the temperature, salinity and pressure in seawater. However, even in the same sea area, the sound speed calculated by different empirical formulas will be quite different. Due to the nonlinearity of the sound velocity profile, when conducting underwater positioning experiments, if the sound velocity value at a certain depth is directly used, the positioning error is often large. At present, the methods of sound velocity correction include weighted average sound velocity method, polynomial fitting method, equivalent sound velocity profile method<sup>[1,2]</sup>, etc.

The above methods are only proposed to compensate the localization error caused by the sound velocity measurement error. Gong Zaixiao<sup>[3]</sup> analyzed the influence of the selection of the reference sound velocity on the sonar passive direction finding results when the beamforming method was used to determine the target Angle. The traditional beamforming method estimates the azimuth information of the target by using the wavepath difference of the received signals between different elements in the receiving array. The correct measurement of the position of the array elements and the correct selection of the reference speed of sound are the prerequisites for accurate azimuth estimation. If the measurement error of the array position is ignored, the accuracy of the reference sound velocity has a great influence on the target Angle estimation result. Gong Zaixiao analyzed the influence of the phase velocity of sound propagation in shallow sea on the direction finding accuracy by using experimental data and numerical simulation, and pointed out that the reference sound velocity in beamforming should be the phase velocity of sound propagation, rather than the sound velocity of seawater. According to the normal mode theory, the phase velocities of normal modes with different numbers are different, and their propagation paths in seawater are also different. In practice, in order to improve the accuracy of the target Angle estimation, it is necessary to select an appropriate phase velocity as the reference sound velocity.

This paper firstly analyzes the influence of the reference sound velocity on the accuracy of passive direction finding of traditional horizontal line array, and analyzes the value of the reference sound velocity based on the normal mode theory. Finally, two methods for calculating the reference sound velocity are proposed: amplitude weighting method and beamforming method, and the applicability of the two methods is analyzed through simulation experiments.

## **2. Array signal model**

The traditional horizontal array passive direction finding method estimates the Angle of the signal by using the delay difference between different elements of the receiving array<sup>[4,5]</sup>. As shown in Figure 1, the horizontal uniform linear array is composed of  $M$  elements, and the element spacing is  $d$ .

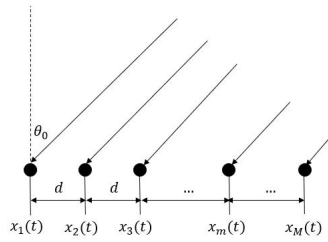


Figure1 Uniform Line Array Model

Assuming that the signal source  $s(t)$  is incident along the direction of Angle  $\theta_0$  with the normal direction of the array, assuming that the first array element in the array is the reference array element, the received signal is:

$$x_1(t) = s(t) + n_1(t) \quad \#(1)$$

The signal received by the M-th array element at time t is:

$$x_m(t) = s(t - \tau_m(\theta_0)) + n_m(t) \quad \#(2)$$

$n_m(t)$  represents the noise received by the M-th array element at time t,  $\tau_m(\theta_0)$  represents the delay of the M-th array element relative to the reference array element:

$$\tau_m(\theta_0) = \frac{(m-1)d \sin(\theta_0)}{c} \quad \#(3)$$

$c$  represents the reference speed of sound. The delay of acoustic signal reaching different array elements is a definite value, assuming that the reference sound speed corresponding to the true incident Angle  $\theta_0$  of the signal is  $c_0$ , if the reference sound speed is  $c_1$  in the actual calculation, and the Angle calculated according to Equation (3) is  $\theta_1$ , then they satisfy the following formula:

$$\frac{\sin \theta_0}{c_0} = \frac{\sin \theta_1}{c_1} \quad \#(4)$$

$$\frac{\sin \theta_0}{c_0} = \frac{\sin(\theta_0 + \Delta\theta)}{c_0 + \Delta c} \quad \#(5)$$

According to Equation (5), the deviation of  $\Delta c$  between the selected reference sound velocity  $c_1$  and  $c_0$  leads to the deviation of  $\Delta\theta$  between the direction finding result and the true value. When the signal is incident at different angles, the direction finding deviation of the selected reference sound speed under different deviations is shown in Figure 2.

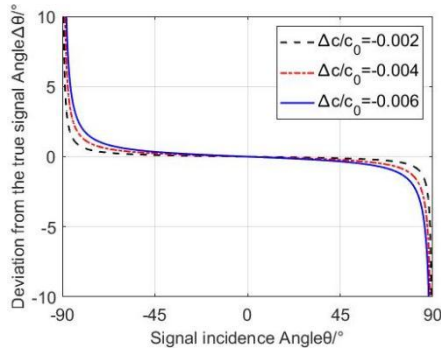


Figure2 Direction finding error caused by reference sound velocity at different incident angles

The direction finding error of horizontal line array is related to the reference sound velocity and the incident Angle of the signal. When the incident direction of the signal is close to the normal direction of the array, the deviation of the reference sound velocity will not lead to obvious direction finding error. When the incident direction of the signal is close to the end-firing direction of the array, the selection deviation of the reference sound velocity will lead to obvious direction finding error, and the larger the deviation of the reference sound velocity, the larger the direction finding error will be.

### 3. The normal mode model

According to the normal mode model, the far field of a point source sound field in shallow water environment solutions for<sup>[6]</sup>:

$$p(r, z) = C \sum_{l=1}^L \Psi_l(z_s) \Psi_l(z) \frac{e^{ik_{rl}r}}{\sqrt{k_{rl}r}} \#(6)$$

$\Psi_l(z)$ ,  $k_{rl}$  represent the eigenfunction and horizontal wave number of l-th normal mode respectively. Suppose that the sound source is a single frequency signal with frequency  $f_0$  and the depth of the sound source is  $Z_s$ , and the receiver array depth is  $Z_r$ . The horizontal distance between the source and the first element of the line array is  $r$ , and the Angle between the source and the normal line direction is  $\theta_0$ . In the far-field condition ( $r \gg d$ ), the horizontal distance between the sound source and the m element can be approximated as:  $r_m = r - (m - 1)dsin\theta_0$ .

The sound pressure value received by the first array element and the M-th array element are respectively:

$$p_1 = C \sum_{l=1}^L \Psi_l(z_s) \Psi_l(z_r) \frac{e^{ik_{rl}r}}{\sqrt{k_{rl}r}} \#(7)$$

$$p_m = C \sum_{l=1}^L \Psi_l(z_s) \Psi_l(z_r) \frac{e^{ik_{rl}r_m}}{\sqrt{k_{rl}r_m}} \#(8)$$

Let  $C_l = C\Psi_l(z_s)\Psi_l(z_r) \frac{e^{ik_{rl}r}}{\sqrt{k_{rl}r}}$ ,  $k = \frac{\omega}{c}$ :

$$p_m \approx \sum_{l=1}^L C_l e^{-ik_{rl}(m-1)dsin\theta_0} \#(9)$$

The vector of sound pressure received by the array element is:

$$p = [p_1, p_2, \dots, p_M]^T = \left[ \sum_{l=1}^L C_l, \sum_{l=1}^L C_l e^{-ik_{r,l} d \sin \theta_0}, \dots, \sum_{l=1}^L C_l e^{-ik_{r,l} (M-1) d \sin \theta_0} \right]^T \quad \#(10)$$

The weighting vector of the traditional beamforming method is:

$$w(\theta) = [1, e^{ikd \sin \theta}, \dots, e^{ik(M-1)d \sin \theta}] \quad \#(11)$$

The beam output function of the traditional beamforming method at the azimuth Angle  $\theta$  is:

$$P(\theta) = w(\theta) * p = \sum_{l=1}^L C_l e^{j \frac{(N-1)d}{2} (k \sin \theta - k_{r,l} \sin \theta_0)} \frac{\sin \frac{Nd(k \sin \theta - k_{r,l} \sin \theta_0)}{2}}{\sin \frac{d(k \sin \theta - k_{r,l} \sin \theta_0)}{2}} \quad \#(12)$$

$$B_l = \sum_{l=1}^L C_l e^{j \frac{(N-1)d}{2} (k \sin \theta - k_{r,l} \sin \theta_0)} \frac{\sin \frac{Nd(k \sin \theta - k_{r,l} \sin \theta_0)}{2}}{\sin \frac{d(k \sin \theta - k_{r,l} \sin \theta_0)}{2}} \quad \#(13)$$

$$P(\theta) = \sum_{l=1}^L B_l \quad \#(14)$$

1) When there is only one normal mode:

When  $k = k_{r,1}$ , the reference sound velocity is the phase velocity of the normal mode,  $P(\theta)$  reaches the peak value at  $\theta_0$ . Therefore, the reference sound velocity used in beamforming should be the phase velocity of sound propagation.

2) When there are multiple normal modes:

If the horizontal wave numbers of each number of normal modes are equal, a  $k$  can be found so that each term  $B_l$  ( $l=1,2,, L$ ) are all the maximum values. When  $k = k_{r,1}$ ,  $P(\theta)$  reaches the peak value at  $\theta_0$ . In fact, the horizontal wave number  $k$  of each normal mode  $k_{r,l}$  is not equal, so you cannot find a  $k$  so that each term  $B_l$  ( $l=1,2,, L$ ) get the maximum value. Therefore, L different output values will be added together to cause  $P(\theta)$  to peak away from the target truth  $\theta_0$ .

## 4. Calculation method of reference sound velocity

This section gives the calculation method of reference sound velocity from two aspects of normal mode amplitude weighting and beamforming.

### 4.1 Amplitude weighting method of normal mode

According to the normal mode theory, the sound pressure at any point in the sound field is the result of the superposition of multiple normal modes excited by the sound source, so:

$$p(r, z) = \sum_{l=1}^L p_l \quad \#(15)$$

The relative energy of each number of normal modes is used as the weighting factor to weight and sum the phase velocity of the corresponding number of signals as the theoretical reference sound velocity  $c_s$  at this position.  $p_l$  is the  $l$ -th normal mode excited by the sound source,  $c_{pl}$  represents the phase velocity of the  $l$ -th normal mode

$$c_s(r, z) = \sum_{l=1}^L \frac{|p_l|^2}{\sum_{l=1}^L |p_l|^2} \cdot c_{pl\#} \quad (16)$$

## 4.2 Beamforming method

Given the target range  $r_s$  and angle  $\theta_s$ , select the sound speed search sequence, such as  $[c_1, c_2, \dots, c_N]$ , because the delay value  $\tau$  between acoustic signals reaching different array elements is fixed, i.e:

$$\frac{\tau}{d} = \frac{\sin(\theta_s)}{c_s} = \frac{\sin(\theta_1)}{c_1} = \frac{\sin(\theta_2)}{c_2} = \dots = \frac{\sin(\theta_N)}{c_N} \quad (17)$$

Traverse the selected sound velocity value to perform beamforming calculation, and obtain the beamforming angle estimation sequence  $[\theta_1, \theta_2, \dots, \theta_N]$ . The sound velocity corresponding to the angle estimation with the minimum angle error of the target is selected as the reference sound velocity at the target distance. The reference sound velocity at different distances can be obtained by changing the horizontal distance between the sound source and the receiving array.

## 5. The simulation results

The simulation parameters is set as follows:

- The sea water depth is 100m.
- The sea water is uniform medium, the sea water density is  $1.0 \text{ g/cm}^3$ .
- The sound speed is 1500m/s. The seabed is a semi-infinite space, the density of the seabed is  $1.6 \text{ g/cm}^3$ , the speed of sound is 1650m/s, and the attenuation coefficient of the seabed is  $0.3 \text{ dB}/\lambda$ .
- The frequency of the sound source ranges from 950Hz to 1050Hz, the depth of the sound source is 25m, and the depth of the receiving array is 50m. The receiving array is an equally spaced horizontal array composed of 48 elements, and the interval of the elements is  $d=0.25\text{m}$ .

The simulation environment is shown in Figure 3.

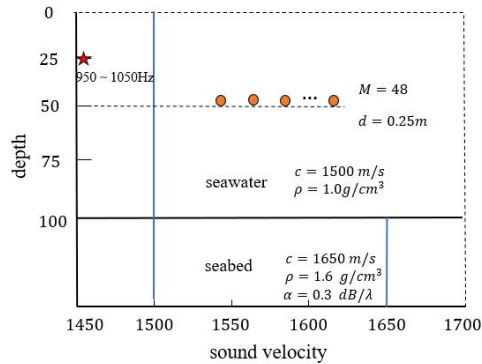


Figure 3 Simulation environment

Figure 4 and Figure 5 respectively show the spectrum of signal Angle estimation at different distances when the incoming direction of the signal is  $20^\circ$  and  $60^\circ$ . The reference sound velocity for calculation is 1500m/s.

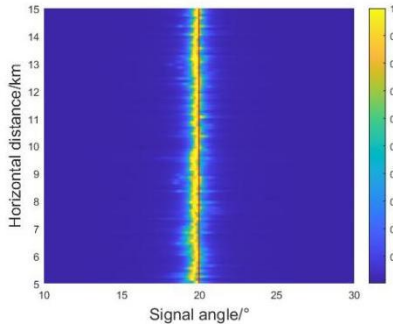


Figure 4 Signal incidence angle of 20°

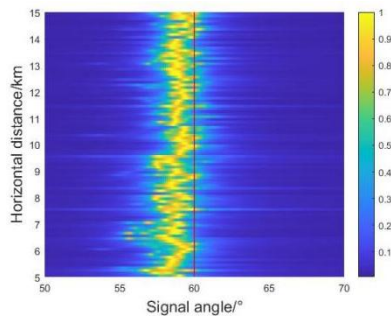


Figure 5 Signal incidence angle of 60°

Comparing the Angle estimation results of the two different signal incident directions, it can be concluded that under uniform hydrological conditions, when the sound source is closer to the receiving array and the direction is more deviated from the normal direction of the array, the deviation of the Angle estimation is larger. When the sound source is farther away from the receiving array, the sound propagation phase velocity at the receiving array decreases gradually and tends to the seawater sound velocity at the depth of the receiving array, and approaches the reference sound velocity gradually. The simulation results show that the reference sound velocity in beamforming is not the sea water sound velocity but the sound propagation phase velocity at the receiving array.

The simulation environment settings remain unchanged. Figure 6 shows the results of the reference sound velocity calculated by using the normal mode amplitude weighting method and the beam forming method as a function of distance.

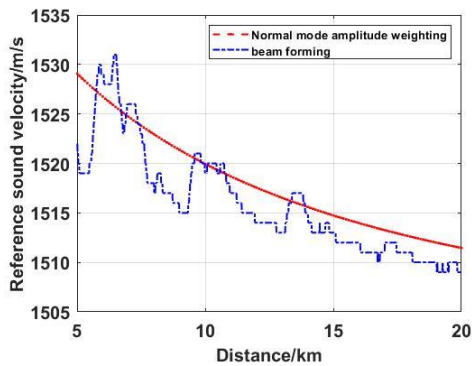


Figure 6 Reference sound velocity

From the simulation results, it can be seen that the reference sound velocity calculated by normal mode amplitude weighting method and beam forming method has the same trend with distance. However, the reference sound velocity value calculated by the beam forming method oscillates in local areas, which is due to the interference of normal modes with different numbers at different distances; The reference sound velocity value calculated by the normal mode amplitude weighting method is generally monotonic. This is because the method does not consider the normal mode interference, but only weights the phase velocity of the corresponding number according to the amplitude of the normal mode with different numbers.

## 6. Conclusion

In this paper, two methods for calculating the reference sound velocity are proposed: normal mode amplitude weighting method and beam forming method. In practical application, the reference sound velocity at different distances can be estimated by using the measured environmental parameters, then the reference sound velocity calculated by the normal mode amplitude weighting method is used for beam forming, combined with the reference sound source aided calibration, and finally the reference sound velocity obtained by the beam forming method is used for target detection.

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