Parameters Estimation for Mathematical Model of Solar Cell

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Abstract: This paper presents, a simplified accuracy solar cell mathematical model is suggested depend on the analysis of single-diode PV cell mathematical model, and afford a parameter determination method depend on two methods Newton-Raphson method (NRM). The voltage of the single-diode is measured numerically based on NRM, then the current and power of the diode is predicted with the variable resistance parameter characteristics are tested under different values of load resistance R from (1-5) Ω under room temperature conditions. The results show that the proposed mathematical model (equation) can quickly and accurately for the PV model I-V and P-V characteristics, which have good methods, and supplies strong support for solar cell system related work.

Keywords: Newton-Raphson Method; Solar Cell Parameters; Mathematical Model; Output Characteristics; Single-diode Equivalent Circuit

1. Introduction

Renewable energy such as solar, wind, wave, and other energy provides new solutions to the growing energy needs. Solar energy is one of the best and most common energies, including photovoltaic technology, which directly generates electricity from solar radiation to DC using semiconductor materials. They are made of monocrystalline or polycrystalline silicon. Due to the exposure of these cells to some external conditions and factors affecting their performance and efficiency, it is necessary to predict their performance under the influence of these conditions. PV cells of any representation of their behavior through mathematical equations, which gives the cell stream as a relationship in its best-known curves characteristics of equations, actually represent the behavior of electrical components circuit equivalent to a cell photovoltaic, which gives the performance its indicators as the value of the maximum possible efficiency cell capacity. The application of the solar cell in satellite (Celestial Mechanics) which the Kepler and Barker equations can be calculated as a nonlinear equation[1-8]. In addition, solar energy is used as an alternative energy to conventional sources such as oil and gas, as it generates clean energy in large quantities and can be transported over long distances. Solar cells can be categorized based on its materials and manufacture process, e.g. monocrystalline, polycrystalline, and thin films, while emergent PV cell can be classified into organic, quantic point; sensitized; CZTS; perovskite and polymer solar cells[9-37]. Another application of solar cell is Laser application [38-40]. It is a branch of mathematics that studies algorithms to solve some mathematical problems related to the use of simple mathematical operations in various areas such as sciences and engineering by using fractional differential equations, optimal control theory and integral equations[41-67].

This paper analyzes the single-diode equivalent circuit model, establishes an accurate engineering mathematical model based on precise mathematical model. Newton- Raphson parameter calculation method
(NRM) is extracted to solve solar cell model, which has an advantage of fast convergence. If we have good initial values, it can accelerate convergence, and precise solution can be got after generally 3-7 times iteration, at the same time meets the requirement of speed and accuracy. By means of the proposed method, this research sets up a simulation model (solar cell model) in MATLAB. The established PV cell model is tested for this method and results show that the (NRM) model has sufficient sensitivity with fewer numbers of iterations in the different values of load resistance in ambient conditions.

2. Analysis of Engineering Mathematical Model

2.1 Single-diode PV Cell Mathematical Model

The method of graphing the relationship between current and voltage produced by a solar cell is the standard form of representation of a cell's output. This graphical representation is called the current-voltage curve (IV Curve). This curve shows an integrated picture of all possible combinations of current and voltage produced by the cell under specific environmental conditions such as radiation, ambient air mass, and temperature. The most commonly used equivalent circuit is a DC source connected in parallel to a semiconductor diode. The equivalent circuit for the simplest solar cell consists of diode and current source connected paralleling, as shown in Figure 1.

![Figure 1. PV-cell equivalent-circuit models: single-diode model.](image)

Current source current is directly proportional to the solar radiation and diode represents PN junction of a solar cell. Equation of ideal solar cell, which represents the ideal solar cell model, is

\[ I_{pv} = I_{ph} - I_D = I_0 \left( e^{\frac{-V_{pv}}{nVT}} - 1 \right) \]  

(1) Ideal PV cell equation

\[ I_D = I_0 \left( e^{\frac{-V_{pv}}{nVT}} - 1 \right) \]  

Diode equation

where:

- \( I_{ph} \) is the photocurrent in (A);
- \( I_0 \) is reverse saturation current in (A);
- \( V_{pv} \) is diode voltage in (V);
- \( V_T \) is thermal voltage = 27.5 \( \pm 26 \) mV at \( T = 25 \) °C Air-Mass = 1.5; n is diode ideality factor (1 < n < 2).

On the other hand, thermal voltage can be determined in the following equation

\[ V_T = \frac{kT}{q} \]  

(3)

where:

- \( k \) is Boltzmann constant= 1.38 \( \times 10^{-23} \) J/K; \( T \) is PV temperature in (K); \( q \) is the charge of electron=1.6 \( \times 10^{-19} \) C.

2.2 Newton-Raphson Method (NRM)

As can be seen, the inclusion of a series resistance makes the solution for current a recurrent equation. One initially applied simple iterative technique only converged for positive currents, but Newton-Raphson method (NRM) used converges much more rapidly, and for both positive and negative current.

NRM can be applied to calculate the voltage of the single-diode \( V_{pv} \) as follows

\[ \frac{df(x)}{dx} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \]  

(4)

From above equation; in order to calculate \( x \)

\[ x_1 - x_0 = \frac{f(x_1) - f(x_0)}{\frac{df(x_0)}{dx_0}} \]  

then \( x_1 = x_0 - \frac{f(x_0)}{\frac{df(x_0)}{dx_0}} \)

generalizing NRM

\[ x_{n+1} = x_n - \frac{f(x_n)}{\frac{df(x_n)}{dx_n}} \]  

(5)

This process is repeated until the convergence criterion is satisfied:

\[ |x_i - x_{i-1}| < \varepsilon \]  

(6)

where \( \varepsilon = 10^{-10} \) is the error tolerance. A graphical representation of Eq. 6 can be seen in Figure 2.

It is apparent that for every approximation \( x_{i-1} \), a better one \( x_i \) of the actual solution \( x \) can be achieved through Eq. 5, \( x_i \) is at the intersection of the function's tangent at \( x_{i-1} \) and axis \( x \).
The following algorithm suggestion for solving Eq. 15 by using NRM (see Figure 3)

**INPUT** initial approximate solution $x_0 = 1$, tolerance $\epsilon$, maximum number of iterations $N$

**OUTPUT** approximate solution $x_{n+1}$

**Step 1:** Set $x = 0$

**Step 2:** while $i \leq x_0$

**Step 3:** Calculate $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for $n = 0, 1, 2, \ldots$

**Step 4:** if $|x_n - x_{n-1}| < \epsilon$; then OUTPUT $x_{n+1}$ and stop.

**Step 5:** Set $n = n + 1; i = i + 1$ and go to Step 2.

**Step 6:** OUTPUT.

![Figure 3. Simulation results.](image)

From Figure 1 $I_{ph} \propto I_{source}$ suppose for 1000 W/m² of isolation $I_{ph} = 10$ A

$$I_{ph} = I_{source}$$

$$I_{D} = I_{S} \times \left(\frac{V_{O}}{n\theta} - 1\right)\theta = I_{S} \times \left(\frac{V_{P}}{n\theta} - 1\right)$$

where $n$ ideally factor $1 < n < 1.5$, $I_{S}$ reverse saturation current $= 10^{-12}$ A. In parallel, $V_{D} = V_{PV} = V$

$$I_{PV} = I_{ph} - I_{D}$$

where $V_{PV} = I_{PV} \times R \rightarrow I_{PV} = \frac{R}{V_{PV}}$

$$I_{PV} = I_{ph} - \frac{R}{V_{PV}}$$

From Eq. 4,

$$I_{ph} - I_{PV} = \frac{R}{V_{PV}}$$

$$I_{PV} = I_{ph} - \frac{R}{V_{PV}}$$

then

$$I_{ph} - I_{D} = \frac{R}{V_{PV}}$$

Substitute Eqs. 7 and 8 into Eq. 14 we get

$$I = 10^{-12} \left(e^{\frac{V_{PV}}{12000}} - 1\right) = \frac{R}{V_{PV}}$$

Eq. 15 can be applied to determine $V_{PV}$ of the diode by using this equation and the first derivative of this equation.

### 3. Results and Discussion

To evaluate and compare the performance of the NRM in catering for the variations of parameters, with the current $I_{PV}$ and voltage $V_{PV}$, of a solar cell, MATLAB software has been developed to extract parameters of this cell. The values of the parameters can be obtained as the voltage $V_{PV}$ have different values; also, with the different values of the load resistance $R$.

The values of $R$ is between 1 to 5 $\Omega$, and using Eq. 16 and the derivative $f(x)$ by means of NRM we can get the Tables from 1 to 5. Table 1 shows the values of the $I_{PV}$ and $P_{PV}$ depending on the extracted values of $V_{PV}$ and using the Eq. 15 based on NRM when the load resistance $R = 1$. Then, the values of $I_{PV}$ and $P_{PV}$ can be calculated.

**Figure 4** shows number of iterations vers the solar cell parameters $I_{PV}$, $P_{PV}$ and $V_{PV}$. From this Figure; on can see that the values of the initial values of the voltage $V_{PV}$ is higher than those of $I_{PV}$

**Table 2** shows the values of the $I_{PV}$ and $P_{PV}$ depending on the extracted values of $V_{PV}$ based on NRM when the load resistance $R = 2$.

**Figure 5** shows number of iterations vers the solar cell parameters $I_{PV}$, $P_{PV}$ and $V_{PV}$. From this Figure; on can see that the values of the initial values of the voltage $V_{PV}$ is higher than those of $I_{PV}$
<table>
<thead>
<tr>
<th>Iterations</th>
<th>R</th>
<th>$X_n$</th>
<th>$X_{n-NRM}$</th>
<th>$I_{pv-NRM}$</th>
<th>$P_{pv-NRM}$</th>
<th>$\epsilon$</th>
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Table 1. Number of iterations using Newton-Raphson method with absolute error $\epsilon$ when $R = 1$.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>R</th>
<th>$X_n$</th>
<th>$X_{pv-NRM}$</th>
<th>$I_{pv-NRM}$</th>
<th>$P_{pv-NRM}$</th>
<th>$\epsilon$</th>
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Table 2. Number of iterations using Newton-Raphson method with absolute error $\epsilon$ when $R = 2$.

Figure 4. No of iterations against $V_{pv}$, $I_{pv}$, $P_{pv}$ at $R = 1$.

Figure 5. No of iterations against $V_{pv}$, $I_{pv}$, $P_{pv}$ at $R = 2$. 
Table 3 shows the values of the $I_{pv}$ and $P_{pv}$ depending on the extracted values of $V_{pv}$ based on NRM when the load resistance $R = 3$.

Figure 6 shows number of iterations versus the solar cell parameters $I_{pv}$, $P_{pv}$ and $V_{pv}$. From this Figure, one can see that the values of the initial values of the voltage $V_{pv}$ is higher than those of $I_{pv}$.

<table>
<thead>
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<th>Table 3. Number of iterations using Newton-Raphson method with absolute error $\varepsilon$ when $R = 3$.</th>
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<td>Iterations</td>
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<tr>
<td>8</td>
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<tr>
<td>9</td>
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</table>

Table 4 shows the values of the $I_{pv}$ and $P_{pv}$ depending on the extracted values of $V_{pv}$ based on NRM when the load resistance $R = 4$.

Figure 7 shows number of iterations versus the solar cell parameters $I_{pv}$, $P_{pv}$ and $V_{pv}$. From this Figure, one can see that the values of the initial values of the voltage $V_{pv}$ is higher than those of $I_{pv}$.

<table>
<thead>
<tr>
<th>Table 4. Number of iterations using Newton-Raphson method with absolute error $\varepsilon$ when $R = 4$.</th>
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Figure 6. No of iterations against $V_{pv}$, $I_{pv}$, $P_{pv}$ at $R = 3$.

Figure 7. No of iterations against $V_{pv}$, $I_{pv}$, $P_{pv}$ at $R = 4$. 

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Table 5 shows the values of the $I_{pv}$ and $P_{pv}$ depending on the extracted values of $V_{pv}$ based on NRM when the load resistance $R = 5$.

Figure 8 shows number of iterations versus the solar cell parameters $I_{pv}$, $P_{pv}$ and $V_{pv}$. From this Figure, one can see that the values of the initial values of the voltage $V_{pv}$ is higher than those of $I_{pv}$.

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<tr>
<th>Iterations</th>
<th>R</th>
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<th>$X_n$-NRM</th>
<th>$I_{pv}$-NRM</th>
<th>$P_{pv}$-NRM</th>
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Table 5. Number of iterations using Newton-Raphson method with absolute error $\varepsilon$ when $R = 5$

4. Conclusion

Mathematical model of the solar cells has strongly nonlinear, multi-parameter that cannot meet needs of practical engineering problems. This research proposed PV cells engineering mathematics model based on single-diode. Newton-Raphson iterative method has been used which is a relatively simple method to calculate the model parameters. Using this method created engineering mathematical model of PV module in MATLAB, test and analysis the PV components such as $I_{pv}$, $P_{pv}$ with various values of $V_{pv}$ in different values of load resistance $R$ from (1-5) $\Omega$. The results show that the proposed method has simple characteristics, for PV cells exhibit good versatility, the model can accurately reflect the output characteristics of PV cells.

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